MATH/STAT 2300 Winter 2016

Assignment 2 - Solutions

Q1/15 Q2/15 Q3/15 Q4/5 Q5/15

1. (a) There are many things that could bring the model into question. For example, the model does not take into account: the weight of the car, the weight of the passengers, the type of car, the status of the brakes, the weather conditions, the road conditions, the age of the driver, the mental state of the driver, and so on. From the plot, the model seems to work well for speeds below 70 miles per hour, but is not good for higher speeds.

(b) In class we developed the following formula for the stopping distance. We can use it to find the stopping distances for the speeds.

$g := x \to 0.054 \cdot x^2 + 1.1 \cdot x;$

$$x \to 0.054\, x^2 + 1.1\, x \quad \textbf{(1)}$$

$g(22.5);$
$g(78);$
$g(120);$

$$52.08750$$
$$414.336$$
$$909.600 \quad \textbf{(2)}$$

These give answers of (i) 52.09 feet, (ii) 414.34 feet and (iii) 909.60 feet.For part (iv) we need to convert from ft/sec to miles/hr:

$v := evalf\left(\frac{50 \cdot 3600}{5280}\right);$
$g(v);$

$$34.09090909$$
$$100.2582645 \quad \textbf{(3)}$$

Thus the stopping distance for (iv) is approximately 100.26 feet.

2. (a)

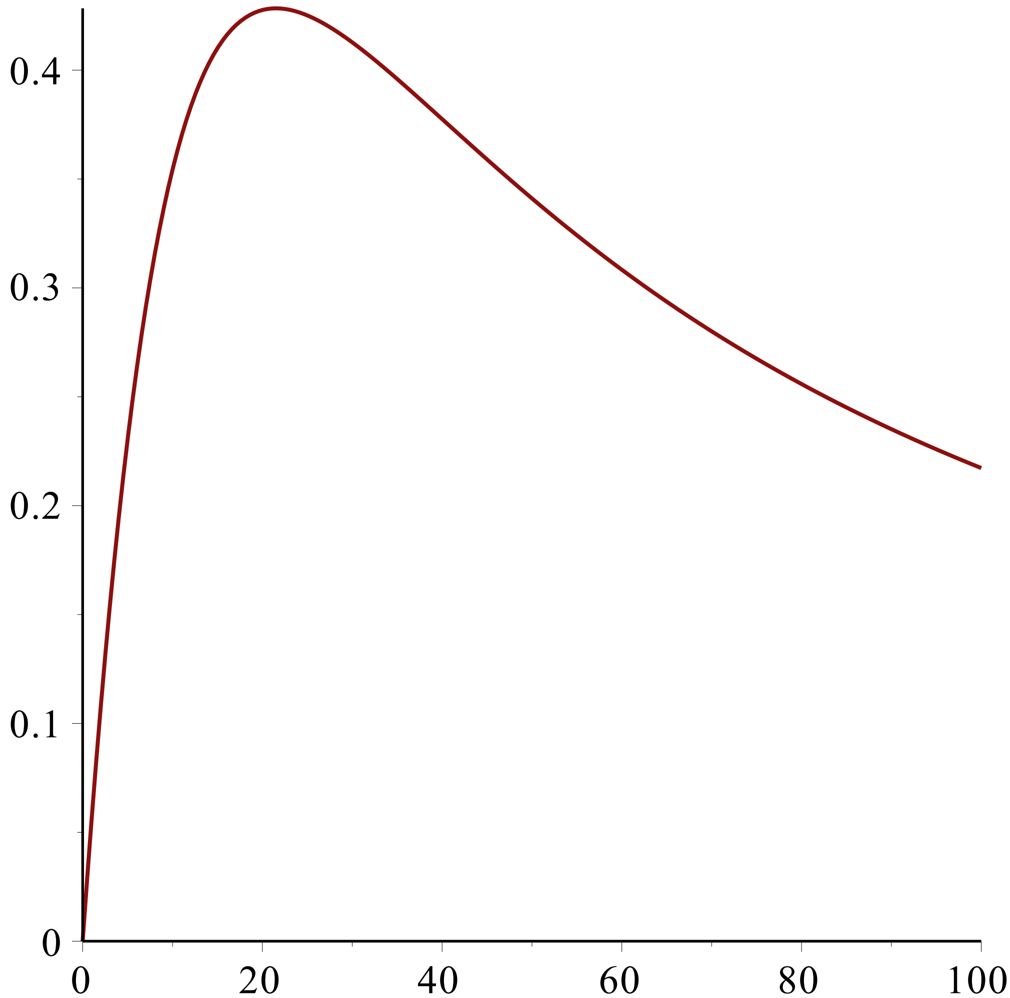
restart;

$f := v \to \frac{\left(\frac{22}{15}\right) \cdot v}{1.1 \cdot v + 0.054 \cdot v^2 + 25};$

$$v \to \frac{22}{15} \frac{v}{1.1\, v + 0.054\, v^2 + 25} \quad \textbf{(4)}$$

Here is a plot of the flow versus the speed (in mph):

$plot(f, 0..100);$



To maximize the flow rate, we use Maple and a bit of calculus:

$g := diff(f(v), v);$
$crit := [solve(g = 0, v)];$

$$\frac{22}{15\left(1.1\,v + 0.054\,v^2 + 25\right)} - \frac{22}{15}\,\frac{v\,(1.1 + 0.108\,v)}{\left(1.1\,v + 0.054\,v^2 + 25\right)^2}$$

$$[-21.51657415, 21.51657415] \quad (5)$$

We can ignore the negative answer. Thus we get a maximum flow rate of 21.93 miles per hour, with a maximum flow rate of

$f(crit[2]);$

$$0.4283751817 \quad (6)$$

which means that about 0.43 of a truck per second passing by any given point on the highway. The corresponding braking distance at this speed is

$subs\left(v = crit[2], 1.1 \cdot v + 0.054 \cdot v^2\right);$

$$48.66823157 \quad (7)$$

feet.

(b) For arbitrary vehicle length b, we get:

```
f := v -> ((22/15)·v)/(1.1·v + 0.054·v^2 + b);
g := diff(f(v), v);
crit := [solve(g = 0, v)];
subs(v = crit[1], 1.1·v + 0.054·v^2);
```

$$v \to \frac{22}{15}\,\frac{v}{1.1\,v+0.054\,v^2+b}$$

$$\frac{22}{15\,(1.1\,v+0.054\,v^2+b)}-\frac{22}{15}\,\frac{v\,(1.1+0.108\,v)}{(1.1\,v+0.054\,v^2+b)^2}$$

$$[4.303314829\sqrt{b},\,-4.303314829\sqrt{b}]$$

$$4.733646312\sqrt{b}+1.000000000\,b \quad \textbf{(8)}$$

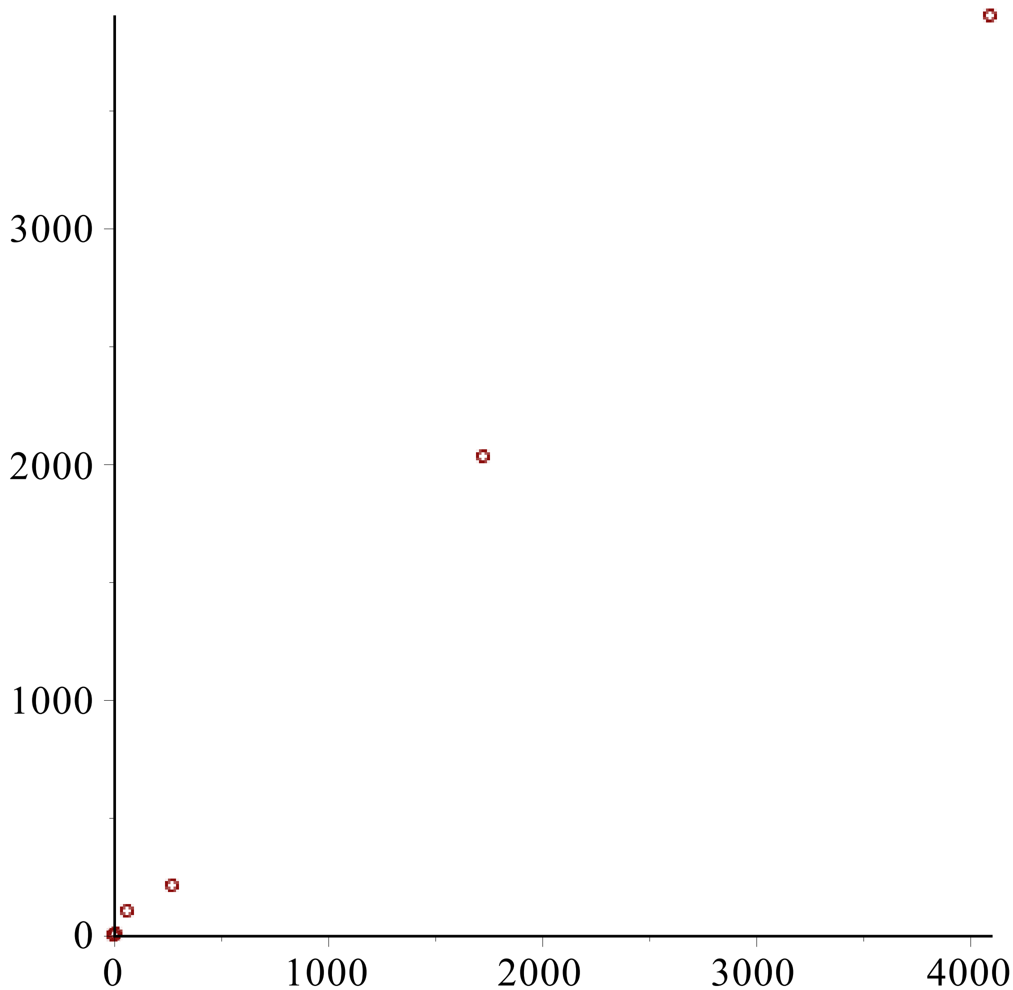
We take the first critical value and see that as b gets larger, the critical point is larger. Plugging this into the stopping distance formula, we get the last line above, which becomes arbitrarily large as b gets large.

3.

```
l := [0.55, 1.0, 2.2, 4.0, 6.5, 12.0, 16.0]:
w := [0.13, 0.64, 5.8, 102, 210, 2030, 3900]:
```

If the hearts of mammals are geometrically similar, then their volumes (v) are proportional to the cube of any characteristic dimension (say the length l of the ventricle), so v is proportional to l^3. Assuming that the hearts are of constant density, we have w is proportional to v, so our model is that w is proportional to l^3. We now plot w versus l^3.

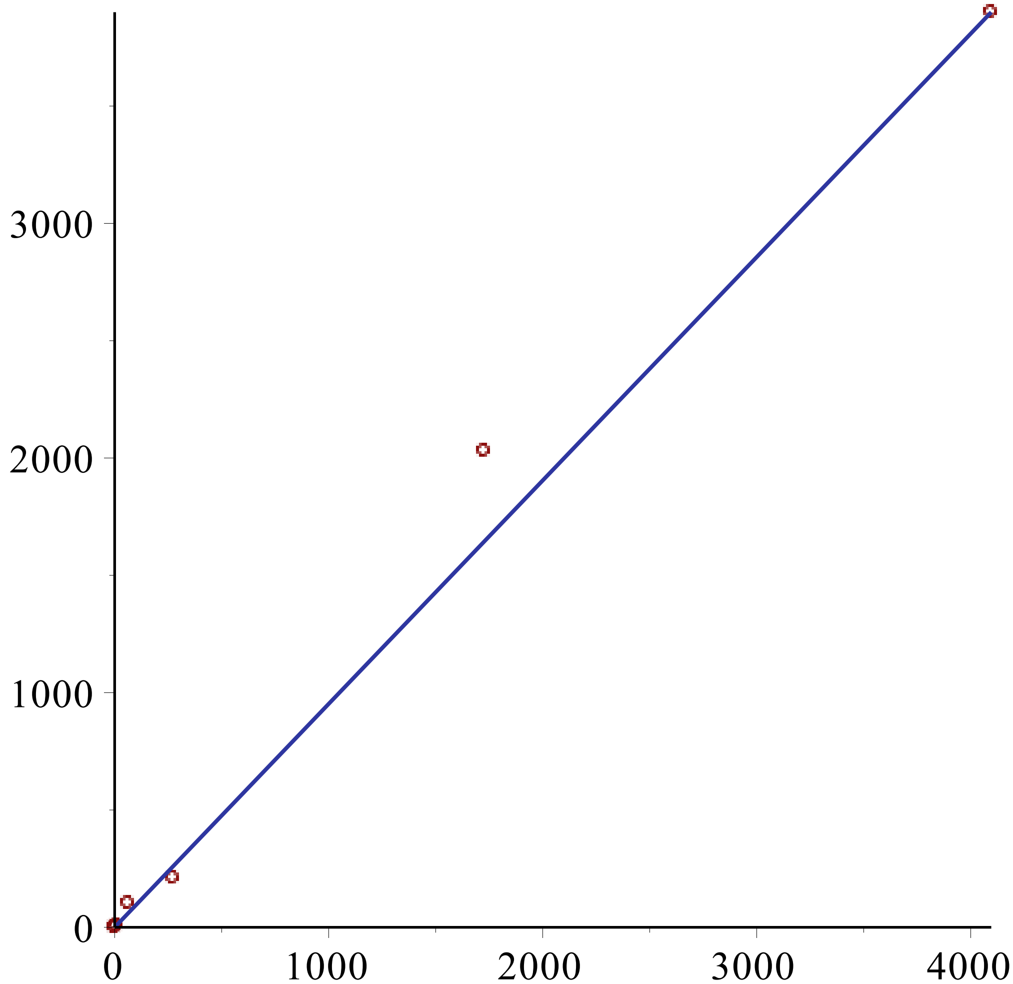
```
p := NULL:
xmax := -infinity:
for i from 1 to nops(w) do
  L := (l[i])^3 :
  p := p, [L, w[i]]:
  if L > xmax then
   xmax := L :
  fi:
 od:
plot([p], 0..xmax, style = point, symbol = circle);
```



It seems perfectly reasonable from the plot that w is proportional to l^3. Let's take the constant of proportionality as the slope between the original and the last point.

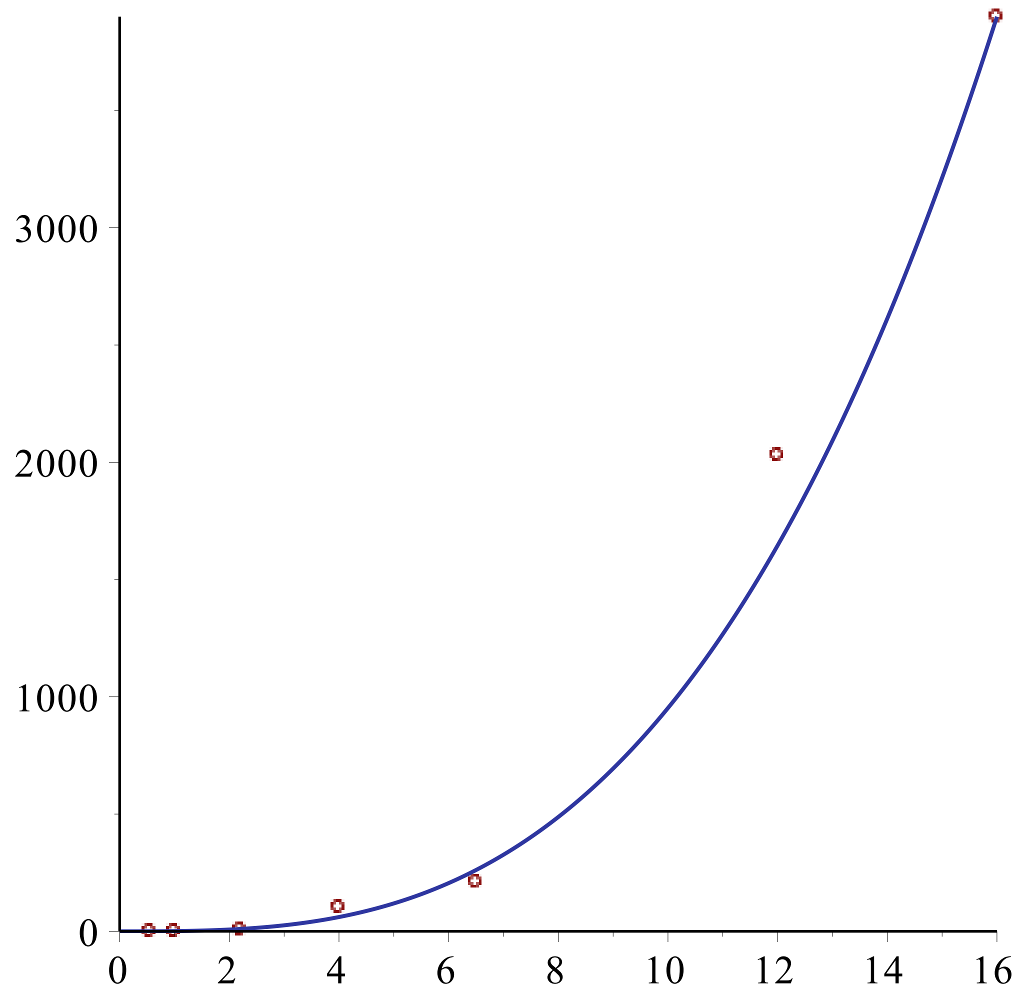
$slpe := \frac{p[7][2]}{p[7][1]}$;
$p1 := plot([p], 0..xmax, style = point, symbol = circle)$:
$modl := x \rightarrow slpe \cdot x$:
$p2 := plot(modl, 0..xmax, colour = navy)$:
$with(plots)$:
$display([p1, p2])$;

0.9521484375



This is a pretty good fit, so we finally look at a plot of the original points along with the model:

```
q := NULL :
xmax := -infinity :
for i from 1 to nops(w) do
  q := q, [l[i], w[i]] :
  if l[i] > xmax then
    xmax := l[i] :
  fi:
 od:
 p3 := plot([q], 0..xmax, style = point, symbol = circle) :
 p4 := plot(x→slpe·x^3, 0..xmax, colour = navy) :
 display([p3, p4]);
```



That's quite a good fit, so we conclude that the hearts of mammals are indeed geometrically similar.

4.

```
with(plots) :
xlist := [1, 2, 3, 4, 5, 6, 7, 8, 9, 10] :
ylist := [6, 15, 42, 114, 311, 845, 2300, 6250, 17000, 46255] :
makePoints := proc(xlist, ylist)
 local pts, n, i :
 pts := NULL :
 n := nops(xlist) :
 for i from 1 to n do
  pts := pts, [xlist[i], ylist[i]] :
 od:
 [pts] :
 end;
```

```
proc(xlist, ylist)                                                    (9)
    local pts, n, i;
```

```
    pts := NULL; n := nops(xlist); for i to n do pts := pts, [xlist[i], ylist[i]] end do; [pts]
end proc
applyFunctionToList := proc(lst, func)
local newlist, n, i :
n := nops(lst) :
 newlist := NULL :
for i from 1 to n do
   newlist := newlist, func(lst[i]) :
 od:
 [newlist];
 end;
```

proc(*lst*, *func*) **(10)**

```
    local newlist, n, i;
    n := nops(lst);
    newlist := NULL;
    for i to n do newlist := newlist, func(lst[i]) end do;
    [newlist]
end proc
```

```
pst1 := makePoints(xlist, ylist);
 f := x→exp(x);
 newlist := applyFunctionToList(xlist, f);
newpoints := makePoints(newlist, ylist);
```

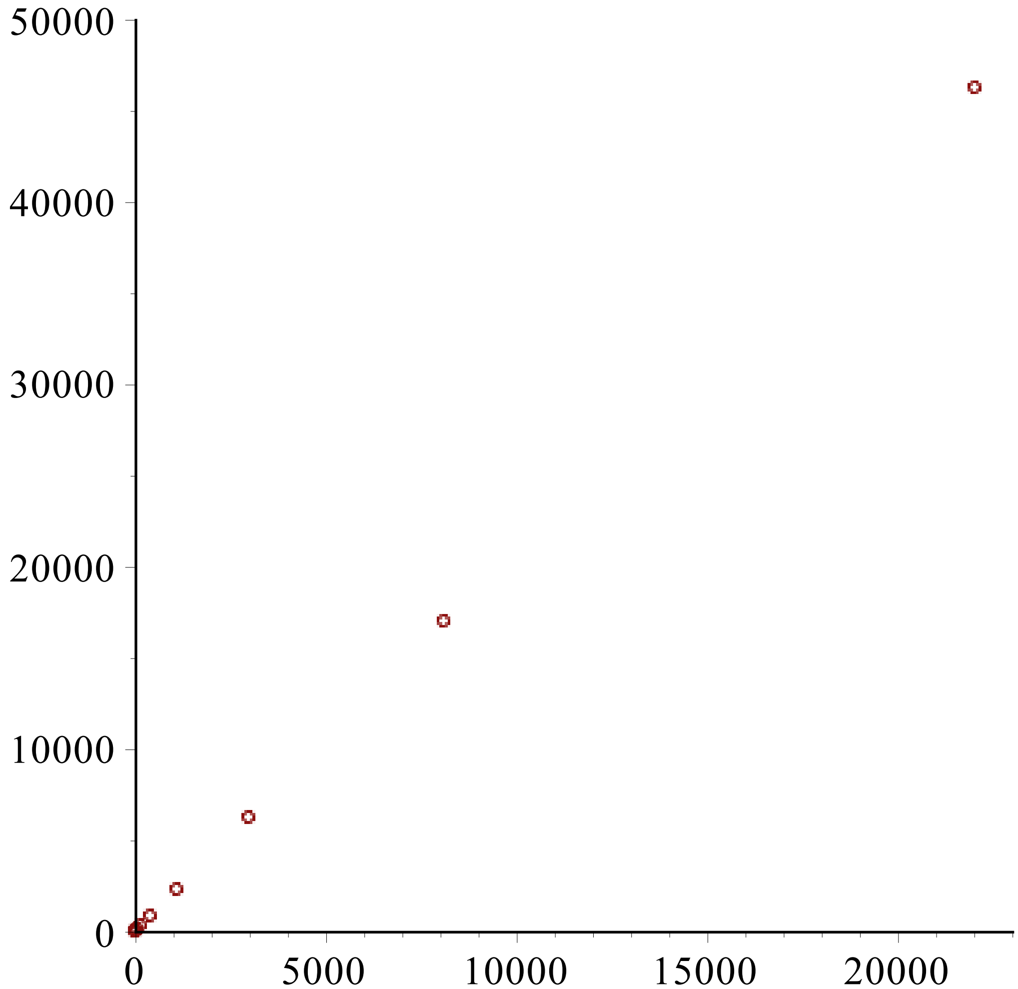
[[1, 6], [2, 15], [3, 42], [4, 114], [5, 311], [6, 845], [7, 2300], [8, 6250], [9, 17000], [10, 46255]]

$$x \rightarrow e^x$$

$$[e, e^2, e^3, e^4, e^5, e^6, e^7, e^8, e^9, e^{10}]$$

$[[e, 6], [e^2, 15], [e^3, 42], [e^4, 114], [e^5, 311], [e^6, 845], [e^7, 2300], [e^8, 6250], [e^9, 17000], [e^{10}, 46255]]$ **(11)**

```
plot(newpoints, 0..23000, 0..50000, style = point, symbol = circle);
```



These points (plotting y versus e^x) look like a straight line through the origin, so we conclude that the data does support y being proportional to e^x.

5. There are many correct answers; your data will decide whether you find the models are approximately correct or not. You should plot your data along with the appropriate model, and state whether the model fits the data. If not, you should explain how the data might be skewed (.e. bad sample, etc.).